PHYSICS 232 Final Examination Name: ____________________

2012/13 Term 2 Student Number: ____________________

Free Response: Write out complete answers to the following questions. Show your work.

(10pts) **1.** Assume that the function $f(x)$ is periodic with period 2π such that $f(x+2\pi) = f(x)$. On the interval $-\pi < x < \pi$, $f(x)$ is given by $f(x) = x/\pi$.

(a) Sketch several periods of $f(x)$. Be sure to include scales for both the $x-$ and $y-$axes of your plot. (2 marks)

(b) Find the Fourier series for this "sawtooth" function. Simplify your answers as much as possible. Write out the first five non-zero terms of the Fourier series. (8 marks)

10 pts

(10pts) **2.** For each of the problems below, assume that you have measured $x \pm \sigma_x$ and $y \pm \sigma_y$. Take A and B to be known constants with negligible uncertainties and n to be an exact integer.

(a) If $f = A\,x + B\,y$, find an expression for σ_f. (2 marks)

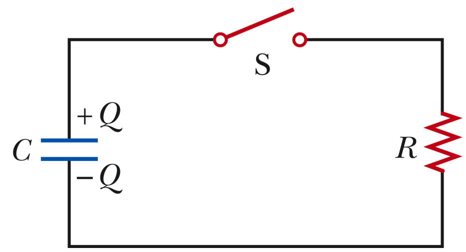
(b) If $f = A\,x\,y^n$, find an expression for σ_f/f. Under what circumstances would the contribution of σ_x to σ_f be negligible? (3 marks)

(c) If $f = A^{B\,x}$, find an expression for σ_f/f. (2.5 marks)

(d) If $f = \ln\left[\sin^n\left(A\,x\right)\right]$, find an expression for σ_f. (2.5 marks)

10 pts

(20[pts]) **3.** Consider the RC circuit shown in the figure. The capacitor is initially charged to voltage V_0. At $t = 0$ the switch is closed and the voltage across the capacitor is recorded as a function of time as shown in the table.



time (s)	V_C (V)
100	3.4 ± 0.2
200	2.3 ± 0.2
300	1.8 ± 0.2
400	1.1 ± 0.2

(a) The voltage across the capacitor is expected to evolve with time according to $V_C = V_0 e^{-t/\tau}$. Linearize this expression such that, using the data given above, the parameters V_0 and τ could be extracted from a linear fit. Clearly explain what you would plot and how the parameters would be extracted from the linear fit. (5 marks)

(b) Using the data given above, create a new table of X and $Y \pm \sigma_Y$. Where a plot of Y vs X is expected to produced a set of linear data as discussed in part (a). For this problem, assume that the uncertainty in time is negligible. (5 marks)

(c) Using the graph paper provided, plot your Y vs X data. Include the σ_Y as error bars. Clearly label your axes and provide a scale for both the $x-$ and $y-$axes. Don't make your plot tiny, use a large portion of the graph paper! Draw a straight line through your data. No calculations are necessary to determine the best-fit line, just use your best judgement. From your line, estimate V_0 and τ. No error estimates are required. (5 marks)

(d) Using your plot (data and line), estimate the value of χ^2. Clearly explain how your are determining χ^2. (5 marks)

20 pts

0 pts

0 pts

(10pts) **4.** The number of flaws in a fibre optic cable follows a Poisson distribution. The average number of flaws in 50 m of cable is 1.2. Recall that, for the Poisson distribution:

$$P_{\mathrm{P}} = \frac{\mu^x}{x!} e^{-\mu}$$

(a) What is the standard deviation in the number of flaws in 50 m of cable? (1 mark)

(b) What is the probability of exactly three flaws in 150 m of cable? (3 marks)

(c) What is the probability of at least two flaws in 100 m of cable? (3 marks)

(d) What is the probability of exactly one flaw in the first 50 m of cable and four or fewer flaws in the next 200 m of cable? (3 marks)

10 pts

Complete any of the **two** remaining problems (5, 6, 7, 8).

Clearly indicate which two problems you wish to be graded by entering two numbers into the table below.

0 pts

(10pts) **5.** Bits are sent over a communications channel in packets of 12. The probability of a bit being corrupted over this channel is 0.1 and such errors are independent.

(a) What is the probability that no more than 2 bits in a packet are corrupted? (4 marks)

(b) If 6 packets are sent over the the channel, what is the average number of packets that contain 3 or more corrupted bits? What is the spread or stand deviation in the number of packets containing 3 or more corrupted bits? (3 marks)

(c) If 6 packets are sent over the channel, what is the probability that at least one packet will contain 3 or more corrupted bits? (3 marks)

10 pts

(10pts) **6.** (a) If repeated measurements of a quantity x are made and the uncertainty in each individual measurement σ is the same (like the oscillation period in Ruchhardt's experiment), then the mean is rather simply estimated via:

$$\mu = \frac{1}{N}\sum_i x_i.$$

Derive an expression for the uncertainty in the mean σ_μ. *Hint*: Use error propagation. (5 marks)

(b) Discuss what happens when the uncertainties in the individual measurements are not the same. How are the mean and the uncertainty in the mean modified? (5 marks)

10 pts

(10pts) **7.** This problem will explore some aspects of fitting functions to datasets. Discuss/comment on the following points:

- What is the origin of χ^2?
- Why is minimizing χ^2 a useful method for extracting best-fit parameters from datasets?
- When doing weighted fits, why is that we assign the weights as $1/\sigma_i^2$ where σ_i is the uncertainty in the i^{th} data point $(x_i, y_i \pm \sigma_i)$?
- Why is it that for models/fit functions that are linear in the unknown parameters we can algebraically determine the best-fit values for the parameters, but for functions nonlinear in the parameters we have to resort to inexact methods such as a grid search?

10 pts

(10pts) **8.** In class we discussed two Monte Carlo methods used to numerically evaluate definite integrals. Pick one of the two methods and outline how it works. Your answer should convey a conceptual understanding of the method and also outline how the method can be implemented. Use diagrams to aid your discussion.

10 pts